Chapter 10 - Tuned Circuits [RESONANCE]

Inductors and capacitors can be combined in series and parallel to form circuits that have the ability to accept or reject signals of particular frequencies. These circuits, which are called tuned circuits, are of great importance in radio.

Reactances in Series

Both capacitors and inductors exhibit reactance in A.C. circuits. The reactance depends on frequency according to the formulae:

XC = -1 / (2 Π f C) ……… **Xc = - 1 / (2 * PI * F * C)**

XL = 2 π f L ……… **XL = 2 * PI * F * L**

When reactances are connected in series – for example, two capacitors or a capacitor and an inductor – then the reactances can be added to give the equivalent reactance of the two reactances in series:

X EQUIV = X1 + X2 + …

For example, suppose we connect two 100 pF (10^-10 F) capacitors in series. At a frequency of 10 MHz (10^6 Hz), the reactance of each of the capacitors individually is:

XC = -1 / (2 **π** f C)
= -1 / (2 * 3,14 * 107 * 10-10)
= -1 / 0,006 28
= -159 Ω

So the equivalent reactance of the two reactances in series is:

XEQUIV = X1 + X2
= -159 + -159
= -318 Ω

Of course there is another way to find this result. Since we have two capacitors of the same value (100 pF) in series, the equivalent capacitance must be half the capacitance of the individual capacitors, or 50 pF (5 * 10-11 F). We can calculate the reactance of this equivalent 50 pF capacitance at 10 MHz (107 Hz) as follows:

XC = -1 / (2 π f C)
= -1 / (2 * 3,14 * 107 * 5 * 10-11)
= -1 / 0,003 14
= -318 Ω

Reactances in Parallel

Similarly, the formula for the equivalent reactance of two reactances in parallel is:

1 / XEQUIV

= 1/X1 + 1/X2 + …

For example, if we take our two 100 pF (10-10 F) capacitors, which each have a reactance of –159 Ω at 10 MHz, and connect them in parallel, then the equivalent reactance is found as follows:

1 / XEQUIV

so

= 1/X1 + 1/X2 + …

= 1/-159 + 1/-159
= -0,012 6

XEQUIV = 1/-0,0126
= -79,5 Ω

Once again this makes sense since the two 100 pF capacitors connected in parallel are equivalent to a single 200 pF (2 * 10-10 F) capacitor, with a reactance at 10 MHz of:

XC = -1 / (2 π f C)
= -1 / (2 * 3,14 * 107 * 2 * 10-10)
= -1 / 0,012 6
= -79,5 Ω

The Series Tuned Circuit

Of course you might well ask, why bother to learn the formulae for reactances in series and parallel if we can calculate the same results using the formulae for capacitors and inductors in series and parallel that we already know? Good question; the answer can be found in the following circuit, which shows an inductor and a capacitor connected in series.

A Series Tuned Circuit

A Series Tuned Circuit

Suppose we want to calculate the equivalent total reactance of these two components at 10 MHz (107 Hz). We can’t use the formula for inductors in series or the formula for capacitors in series, since the circuit contains one of each. So instead we must calculate the individual reactances of each component at a frequency of 10 MHz, and then use the formula for reactances in series.

The reactance of the inductor is found as follows:

XL = 2 π f L
= 2 * 3,14 * 107 * 6,5 * 10-6
= 408 Ω

The reactance of the capacitor is given by:

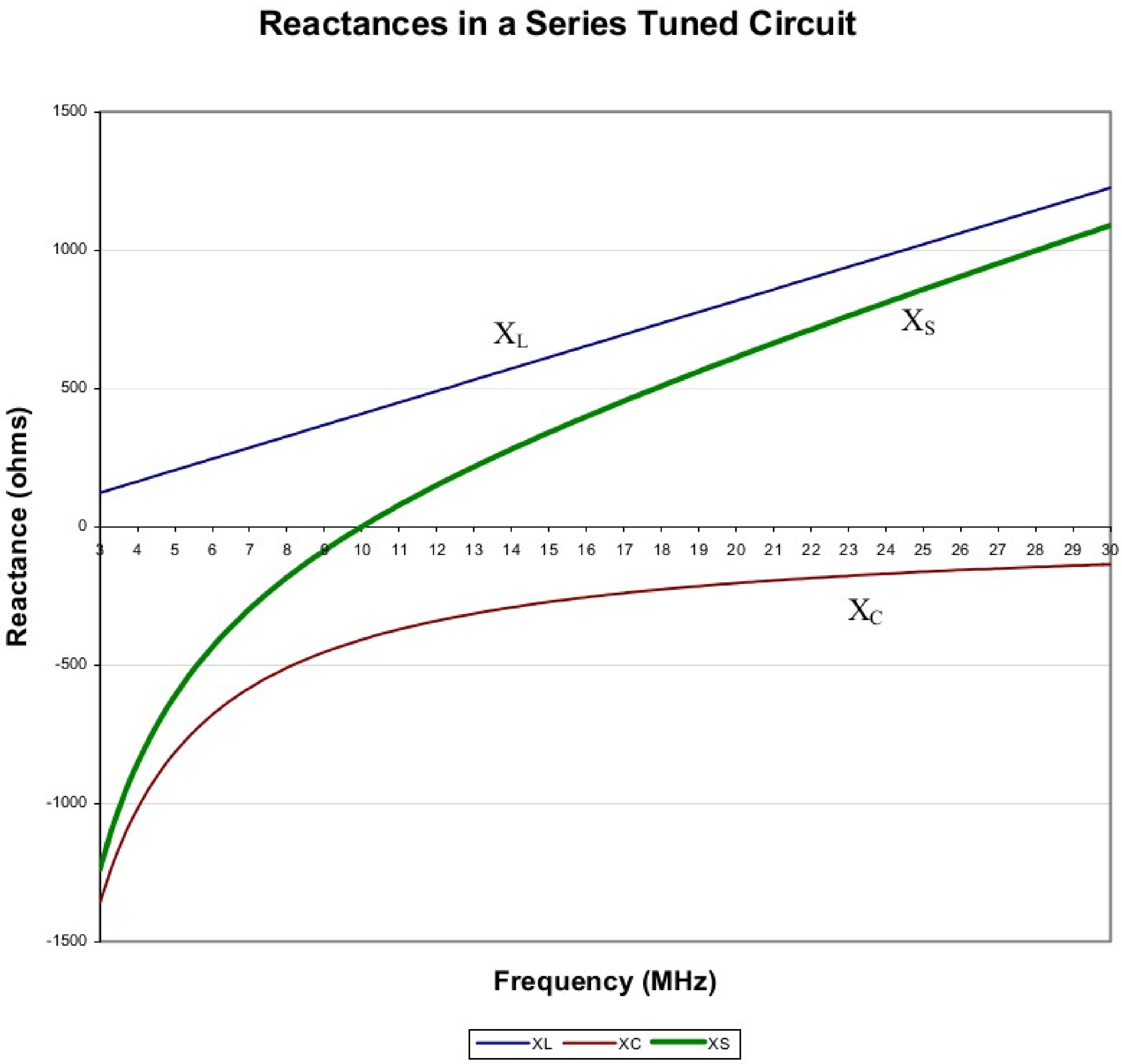
XC = -1 / (2 π f C)
= -1 / (2 * 3,14 * 107 * 39 * 10-12)
= -1 / 0,006 908
= -408 Ω

So the combined reactance of the inductor and capacitor in series at 10 MHz is

XEQUIV = XL + XC
= 408 – 408
= 0Ω

That's right – zero! The capacitor has reactance, and the inductor has reactance, but at this frequency (10 MHz) the positive reactance of the inductor exactly cancels out the negative reactance of the capacitor, leaving no reactance at all! The frequency at which the positive and negative reactances cancel out is known as the resonant frequency of the circuit. The circuit itself is called a series resonant circuit or a series tuned circuit.

Since the reactance of the inductor increases with frequency, while the reactance of the capacitor decreases with frequency (if you forget about the minus sign), this cancelling out will only happen at one specific frequency. At any other frequency, the circuit will exhibit either inductive (positive) or capacitive (negative) reactance. The graph below shows the inductive reactance XL (which is always positive), capacitive reactance XC (always negative) and the combined reactance of the series circuit XS. As you can see, the combined reactance is negative (capacitive) below the resonant frequency of 10 MHz, and positive (inductive) above the resonant frequency.



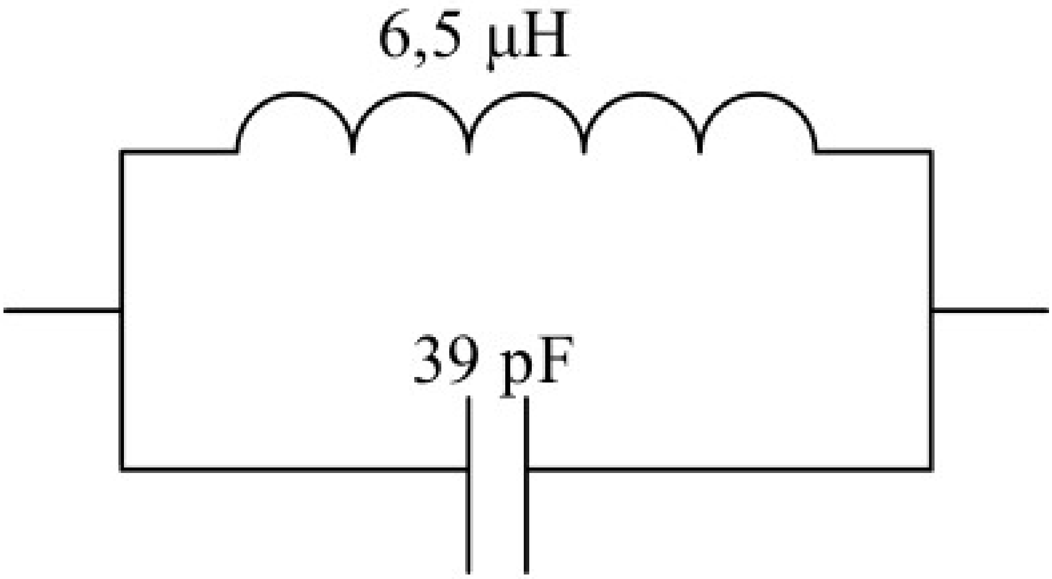
Reactances in a Series Tuned Circuit

The series tuned circuit is very useful in radio electronics as the low reactance near the resonant frequency means that current can easily flow in the circuit near this frequency; while the high reactance at other frequencies will oppose the flow of current at frequencies other than the resonant frequency. In this way, a series tuned circuit can be used to accept signals with frequencies near the resonant frequency, while rejecting other signals.

The Parallel Tuned Circuit

Having seen the strange and interesting behaviour we get when we connect an inductor and capacitor in series naturally raises the question of what would happen if we were to connect them in parallel. To save us unnecessary calculations, we may as well choose the same values

– L = 6,5 µH and C = 39 pF.



A Parallel Tuned Circuit

A Parallel Tuned Circuit

Once again we will calculate the combined reactance at 10 MHz – since this was the resonant frequency for the series tuned circuit, perhaps it will also show some interesting behaviour in this parallel tuned circuit.

From the formula for reactances in parallel, we know that

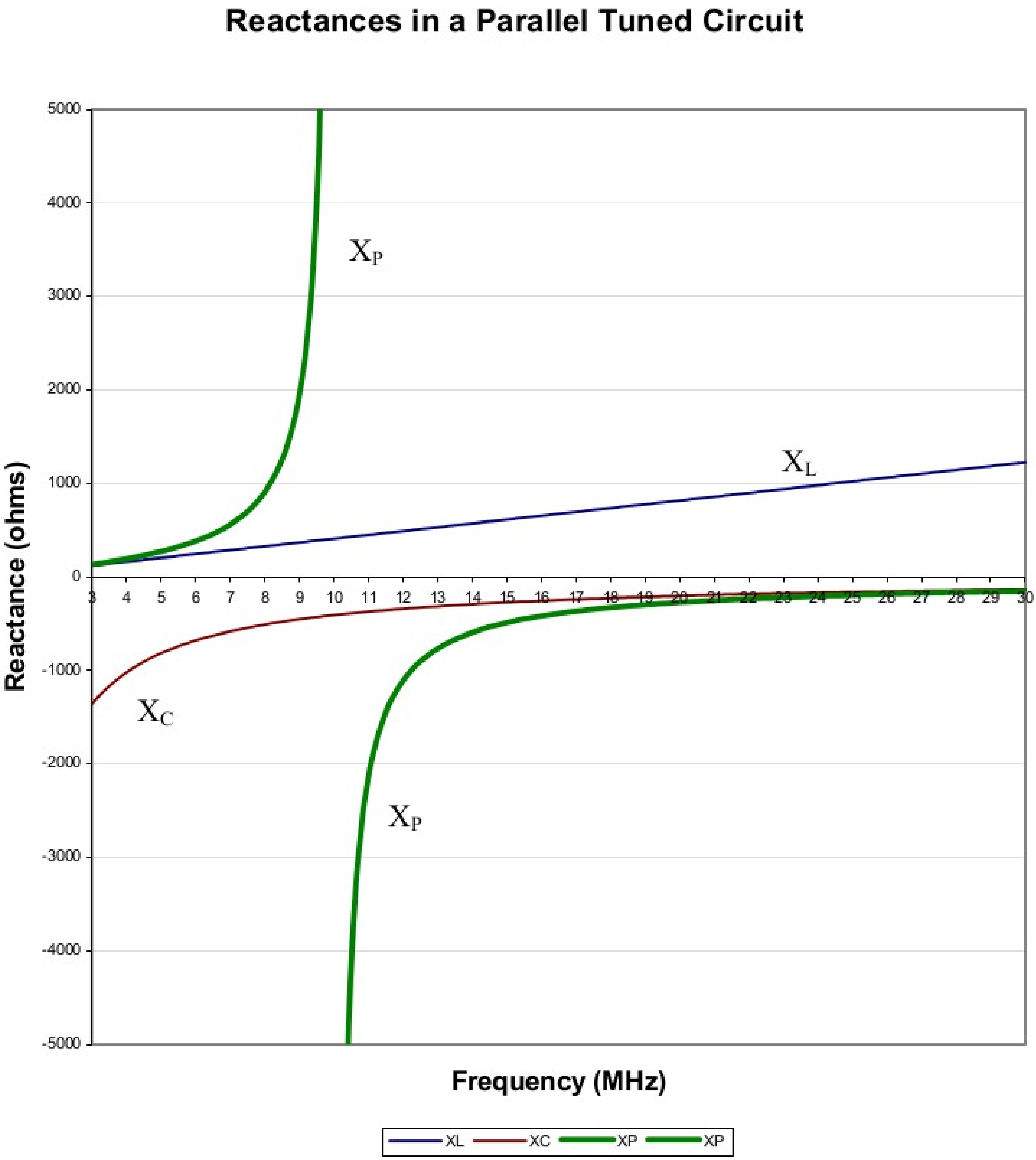
1 / XEQUIV

so

= 1/XL + 1/XC
= 1/408 + 1/-408
= 0,00245 – 0,00245
=0

XEQUIV = 1/0
= ???? [!!!] Not really, just very high.

What has happened here? Once again the positive inductive reactance has cancelled out the negative capacitive inductance, but this time it has left the zero in the denominator (bottom) of a fraction, which means that the result is undefined. However if we plot a graph showing the reactances for a range of frequencies, we will understand what is happening better.



Reactances in a Parallel Tuned Circuit

Once again the inductive reactance is always positive, while the capacitive reactance is always negative. This time however the combined reactance of the tuned circuit starts slightly positive (inductive) and rapidly gets more and more positive as the resonant frequency is approached. However at the resonant frequency it instantaneously transitions from being a very high positive (inductive) reactance to being very high negative (capacitive) reactance.

No wonder the exact value at resonance is undefined.

As a result, a parallel tuned circuit has a high reactance near resonance while its reactance is small away from the resonant frequency. This means that a parallel tuned circuit can be used to block signals near its resonant frequency, while allowing signals of other frequencies to pass relatively easily.

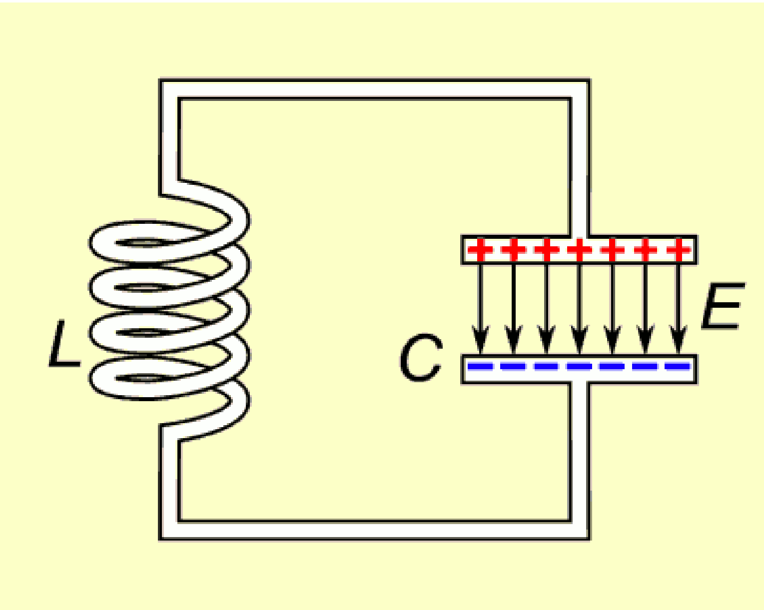
Circulating Current in a Parallel Tuned Circuit

A parallel tuned circuit has two components that are capable of storing energy. The inductor stores energy in its magnetic field; and the capacitor stores energy in the electric field between its plates. At resonance, energy is constantly being transferred from the capacitor to the inductor and back again.

As the capacitor charges up, a voltage develops between its plates. This voltage causes a current to flow through the inductor, which generates a magnetic field. As the capacitor discharges the voltage across its plates drops, which tends to reduce the current flowing through the inductor. However an inductor will resist any attempt to change the current flowing through it. The magnetic field of the inductor collapses, inducing a potential difference into the inductor that acts to keep the current flowing in the same direction as it was before. This current flow now charges the capacitor up again, but with the opposite polarity to before. As the capacitor charges a voltage develops across its plates. This voltage causes current to flow through the inductor in the reverse direction, which generates a magnetic field, and so on.

So the parallel tuned circuit acts somewhat like a **pendulum**, continually transferring energy between two different forms. (In the pendulum, these forms are the potential energy when the pendulum is stationary at the top of its arc, and the kinetic energy when the pendulum is moving at maximum speed at the bottom of its arc).

One result of this is that the circulating current that flows in a parallel tuned circuit – that is, the current flowing around the circuit containing the capacitor and the inductor – can be much larger than the current that the parallel tuned circuit is drawing from the rest of the circuit. In practical circuits, it is not uncommon to have a **circulating current** that is 100 times the size of the current that the parallel tuned circuit is drawing from the external circuit.



Calculating the Resonant Frequency

We have seen that in both a series tuned circuit and a parallel tuned circuit, something interesting happens at the resonant frequency which is where the reactance of the capacitor and inductor have the same magnitude (value) but one is positive and the other is negative so they cancel each other out. We can derive a formula for the resonant frequency as follows:

At resonance, the magnitude of the capacitive and inductive reactances are equal, so

$$2 \pi f L = 1 / (2 \pi f C)$$

so

$$f2$$

and

$$f = 1 / (4 \pi^2 L C)$$
$$= 1 / (2 \pi \sqrt{LC})$$

You do not need to know the derivation, but you should be able to apply the result. For example, let us calculate the resonant frequency of a series or parallel circuit consisting of a 6,5 µH inductor and a 39 pF capacitor:

$$f = 1 / (2 \pi \sqrt{LC})$$

$$= 1 / (2 * 3,14 * \sqrt{6,5 * 10-6 *39 * 10-12})$$
$$= 1 / (6,28 * \sqrt{253,5 * 10-18})$$
$$= 1 / (6,28 * 1,59 * 10-8)$$
$$= 1 / 10-7$$
$$= 107 \text{ Hz}$$
$$= 10 \text{ MHz}$$

Circuit Losses and the Quality Factor [Q]

The discussion so far has ignored circuit losses. For example, all practical inductors have some resistance as well as their inductance, and capacitors also have some losses although these are typically negligible compared to the losses caused by the resistance of the inductor.

The effect of these losses is that in a practical series tuned circuit, although at resonance the reactance would be zero, there would still be some small resistance. In a parallel tuned circuit, the effect of circuit losses is to limit the reactance at resonance to a high but finite value, rather than being completely undefined (or "infinite") as predicted by the maths.

The extent of circuit losses is expressed by a number called the **"Quality Factor"**, or **"Q Factor"** or just the "Q" of the tuned circuit. A high Quality Factor means low circuit losses, while a low Qualify Factor means high circuit losses. The quality factor is defined as the reactance of either the inductor or the capacitor at resonance divided by the circuit resistance.

So

$$Q = XL / R$$
$$= -XC / R$$

(The minus sign in the second line is just to take account of the fact that capacitive reactance is itself negative and ensure that we come up with a positive Q). The Quality Factor of practical tuned circuits is typically between 50 and 200.

The Quality Factor is related to two other properties of the tuned circuit:

1. The ratio of circulating current in a parallel tuned circuit to the current drawn by the tuned circuit is the same as the Q factor. So in a parallel tuned circuit with a Q of 100, the circulating current will be 100 times greater than the current drawn from the rest of the circuit.
2. The selectivity of the circuit – that is, its ability to allow desired signals through while blocking undesired signals. The greater the Q of the tuned the circuit, the greater its selectivity.

Summary
The series tuned circuit has a low reactance near its resonant frequency, and a high reactance at other frequencies. Series tuned circuits are often used to allow signals near the resonant frequency to pass, while blocking signals at other frequencies.

The parallel tuned circuit has a high reactance near its resonant frequency, and a low reactance at other frequencies. Parallel tuned circuits are often used to block signals near the resonant frequency, while allowing signals at other frequencies to pass.

The resonant frequency of a series or parallel tuned circuit may be calculated as

$$\mathbf{f = 1 / (2 \pi \sqrt{LC})}$$

The Quality Factor (“circuit Q”) is defined as the reactance of either the inductor or the capacitor at resonance divided by the circuit resistance. A tuned circuit with a high Q is more selective than a tuned circuit with a low Q.

The circulating current in a parallel tuned circuit may be many times the current drawn by the tuned circuit. The ratio between the circulating current and the current flowing into the tuned circuit is the same as the Quality Factor.